Vibration Error Suppression of SINS Based on ATFPF and Improved Second-order Coning Compensation Algorithm

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Abstract: The error caused by environmental vibration restricts the measurement accuracy of strapdown inertial navigation system (SINS). In order to reduce the influence of vibration on SINS, a method based on adaptive time-frequency peak filtering (ATFPF) and improved second-order coning compensation algorithm is proposed. In this paper, firstly, the implementation of ATFPF is studied and compared with wavelet filtering algorithm. The results show that the ATFPF algorithm can achieve better filtering effect for gyro output signal. Secondly, the traditional coning compensation algorithm is improved, and the expression of two-order three-sample coning compensation algorithm is derived. Finally, the result of simulation shows that this method is superior to the traditional three-sample coning algorithm in suppressing the vibration error. The error mean of attitude can be decreased by about one order of magnitude.

1. Introduction

Vibration from surrounding environment is an important error source of SINS. When the carrier adopts the strapdown inertial system for navigation, the dynamic error caused by its own line vibration and angular vibration brought by environment is an important factor that restricts the measurement accuracy.

In this paper, a method combining the adaptive time-frequency peak filtering (ATFPF) and the improved second-order coning compensation algorithm for the gyro signal is proposed to deal with the vibration noise. The simulation results show that the vibration noise is suppressed and the navigation precision of SINS is improved.

2. Adaptive Time-frequency Peaking Filtering Algorithm

Time-frequency peak filtering (TFPF) is a signal enhancement algorithm based on time-frequency analysis theory. TFPF has been widely applied in the field of signal processing. The basic principle of TFPF is to convert a signal submerged under noise into an instantaneous frequency (IF) of a constant amplitude FM signal by frequency modulation. TFPF uses a pseudo Wigner-Ville distribution (PWVD) to reduce the bias of estimation of instantaneous frequency and

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improve the filtering accuracy. A standard TFPF algorithm has 3 steps:

1) in this paper, the original signal is the output signal of inertial measurement unit (IMU), whose amplitude is scaled to a certain range to avoid aliasing of the encoded signal.

2) the scaled noise-containing output signal is encoded by a frequency modulation method and converted into a frequency-modulated analytical signal with a constant amplitude.

3) the IF is estimated by the PWVD peak of the coded signal and restored to the original signal.

Through inverse scaling operation, estimated signal can be restored to the output signal of IMU.

This paper uses PWVD to process the gyro output signal, and the window length of PWVD has an effect on the suppression of noise and estimation of the instantaneous frequency, so in order to get the best filtering performance, a TFPF algorithm with variable window length is proposed to adjust the window length in real time to adapt to the fluctuation of signal. In this paper, ATFPF is adopted, which can select the appropriate window length for PWVD operation on the basis of the TFPF algorithm in real-time and improve the accuracy of the algorithm.

In order to study the effect of ATFPF algorithm, two wavelet filtering algorithms (db4 wavelet and sym2 wavelet) are used as comparison. We filter a set of gyro static output data (sampling frequency 200Hz, duration 320s), as shown in Fig. 1 and Table 1.

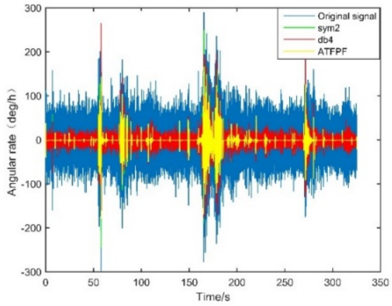

Fig. 1 Comparison of static gyro output signal before and after filtering

Table 1 Comparison of static gyro output signal before and after filtering

	Mean（deg/h）	Standard deviation（deg/h）	SNR (dB)
Original signal	-1.6314	38.9661	-----
Sym2	-1.6331	17.6025	1.1194
Db4	-1.6332	17.6015	0.9580
ATFPF	-1.6309	14.3795	3.0136

The mean values of the signal before and after filtering are close. The standard deviation of the signal filtered by ATFPF is 63% lower than that of the original signal and 18% lower than that of other wavelet filtering algorithms. And its SNR is 2dB higher than that of other wavelet filtering algorithms. This illustrates that ATFPF filtering algorithm can obtain better filtering effect than wavelet filtering for static gyro output.

3. Second-order Coning Compensation Algorithm

The study of high-precision rotation vector algorithms for conical motion began in the 1970s. Bortz first proposed the concept of a rotation vector to compensate for the non-commutative error of the traditional quaternion algorithm. The following scholars put forward various conic algorithms on this basis. including Miller's three-sample coning compensation algorithm:

$$\Phi = \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 + \frac{9}{20}\Delta\theta_1 \times \Delta\theta_3 + \frac{27}{40}\Delta\theta_2 \times (\Delta\theta_3 - \Delta\theta_1) \quad (1)$$

where $\Delta\theta_i$ is the angular increment of each sample in the update period of the algorithm.

According to the references, the truncation error of the algorithm is

$$\bar{\delta}_T = \sin^4\frac{\alpha}{2}\left[\frac{1}{15}(\Omega h)^5 - \frac{29}{3645}(\Omega h)^7 + \frac{4678}{10333575}(\Omega h)^9 + \cdots\right] \tag{2}$$

The constant drift errors of the algorithm is

$$\Phi_\varepsilon = \frac{\alpha^2(\Omega\,\mathrm{h})^7}{204120h} \tag{3}$$

where Ω is the angular frequency of conical motion, h is the update period and α is the semi-cone angle of conical motion.

From equation (2) and (3), the truncation error is much larger than the constant drift in some cases. For example, $\alpha=1^\circ$, $\omega=6\ rad/s$ and $h=0.01s$. Therefore, the high accuracy coning algorithm should not neglect compensation for the truncation error term. For the three-sample coning algorithm, $\bar{\delta}_T$ should be compensated.

According to Bortz's second-order rotational vector differential equation:

$$\Phi=\Delta\theta+\frac{1}{2}\int_{t_{m-1}}^{t_m}\Phi\times\omega dt+\frac{1}{12}\int_{t_{m-1}}^{t_m}\Phi\times(\Phi\times\omega)dt \tag{4}$$

$$\Phi=\Delta\theta+\delta\Phi,\delta\Phi\approx\frac{1}{2}\int_{t_{m-1}}^{t_m}\Delta\theta\times\omega dt \tag{5}$$

Substituting equation (5) into (4):

$$\Phi=\Delta\theta+\frac{1}{2}\int_{t_{m-1}}^{t_m}(\Delta\theta\times\omega)dt+(\frac{1}{4}\int_{t_{m-1}}^{t_m}(\int_{t_{m-1}}^{t_m}\Delta\theta\times\omega dt)\times\omega dt+\frac{1}{12}\int_{t_{m-1}}^{t_m}\Delta\theta\times\left(\Delta\theta\times\omega\right)dt)=\Delta\theta+\delta\Phi+\delta\delta\Phi \tag{6}$$

where $\delta\delta\Phi$ represents two order conic compensation and it should compensate the truncation error according to the above analysis.

In one update cycle, the gyro angle increment can be approximated by a cubic polynomial:

$$\Delta\theta=a\left(t-t_{m-1}\right)+b\left(t-t_{m-1}\right)^2+c\left(t-t_{m-1}\right)^3,t\in\left(t_{m-1},t_m\right) \tag{7}$$

Substituting equation (7) into the $\delta\delta\Phi$ term of (6) and after simplification and summarization,

$$\delta\delta\hat{\Phi}=\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k=j+1}^{N}l_{ijk}\Delta\theta_i\times\left(\Delta\theta_j\times\Delta\theta_k\right),N=3 \tag{8}$$

According to the references,

$$\left[\Delta\theta_i\times\left(\Delta\theta_j\times\Delta\theta_k\right)\right]=-\frac{16}{3}\Omega h\sin^2\alpha\sin^2\left(\frac{\Omega h}{6}\right)\sin\left(\frac{k-j}{6}\Omega h\right)\sin\left(\frac{2i-k-j}{6}\Omega h\right) \tag{9}$$

the value of equation (9) is only related to $|k-j|$ and $|2i-j-k|$.

$$\delta\delta\hat{\Phi}=l_1\left[\Delta\theta_1\times\left(\Delta\theta_1\times\Delta\theta_2\right)\right]+l_2\left[\Delta\theta_3\times\left(\Delta\theta_1\times\Delta\theta_2\right)\right]+l_3\left[\Delta\theta_1\times\left(\Delta\theta_1\times\Delta\theta_3\right)\right] \tag{10}$$

We perform Taylor series expansion on Ωh in the equation (10)

$$\begin{aligned}\delta\delta\hat{\Phi}\approx\sin^4\frac{\alpha}{2}\{&l_1\left[\frac{4}{243}(\Omega h)^5-\frac{2}{6521}(\Omega h)^7+\frac{1}{393660}(\Omega h)^9+\cdots\right]+l_2\left[-\frac{4}{81}(\Omega h)^5+\frac{2}{729}(\Omega h)^7-\frac{3}{393660}(\Omega h)^9+\cdots\right]\\&+l_3\left[\frac{16}{243}(\Omega h)^5-\frac{20}{6521}(\Omega h)^7+\frac{2}{32805}(\Omega h)^9+\cdots\right]\}\end{aligned} \tag{11}$$

Comparing equation (11) with equation(2), we can obtain

$$\frac{4}{243}l_1-\frac{4}{81}l_2+\frac{16}{243}l_3=\frac{1}{15},-\frac{2}{6521}l_1+\frac{2}{729}l_2-\frac{20}{6521}l_3=-\frac{29}{3645},\frac{1}{393660}l_1-\frac{23}{393660}l_2+\frac{2}{32805}l_3=\frac{4678}{10333575} \tag{12}$$

We solve the system of equations and get an approximate solution

$$l_1\approx\frac{-14153}{73},l_2\approx\frac{13742}{75},l_3\approx\frac{11401}{61} \tag{13}$$

Therefore, the second-order three-sample coning compensation algorithm can be expressed as

$$\begin{aligned}\Phi=&\Delta\theta_1+\Delta\theta_2+\Delta\theta_3+\frac{9}{20}\Delta\theta_1\times\Delta\theta_3+\frac{27}{40}\Delta\theta_2\times\left(\Delta\theta_3-\Delta\theta_1\right)\\&+\frac{-14153}{73}\Delta\theta_1\times\left(\Delta\theta_1\times\Delta\theta_2\right)+\frac{13742}{75}\Delta\theta_3\times\left(\Delta\theta_1\times\Delta\theta_2\right)+\frac{11401}{61}\Delta\theta_1\times\left(\Delta\theta_1\times\Delta\theta_3\right)\end{aligned} \tag{14}$$

According to the above analysis, equation (14) can compensate for drift and truncation errors (equation (3) and (2)) theoretically, and improve the accuracy of attitude calculation.

4. Simulation Experiment

The trajectory generator generates an aircraft trajectory with a duration of 76 seconds. The initial position of the aircraft is 160 degrees east longitude and 40 degrees north latitude and its height is 150 meters. The sampling frequency of system is 100Hz. The track is shown in the Fig. 2.

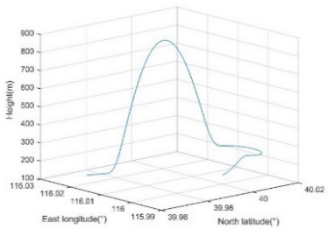

Fig. 2 Trajectory chart of aircraft

Random noise, sinusoidal noise at certain frequencies and pink noise are added to the gyroscope signal from the trajectory generator. The ATFPF and improved three-sample coning compensation algorithm described above are used to perform attitude calculation on the noisy gyroscope and accelerometer signals, and the results are compared with those of the trajectory generator.

Taking the output of gyro x-axis as an example, its original signal, noise-added signal, and filtered signal after ATFPF are shown in Fig. 3.

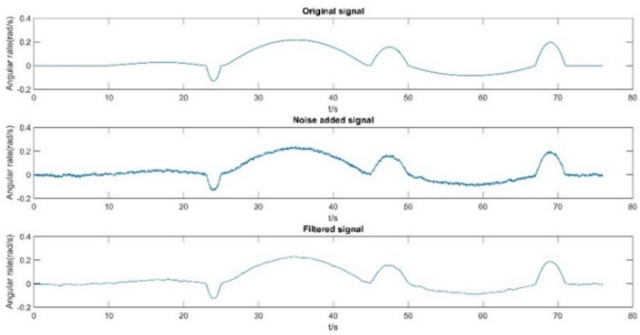

Fig. 3 Comparison of gyro X axis before and after filtering

The signal-to-noise ratio (SNR) of the filtered signal is 32.5905 dB and SNR of pre-filtering signal is 25.4513 dB and SNR is increased by 2dB. We can conclude that the SNR of filtered signals has been improved.

We use different combinations of algorithms to perform attitude calculation, and the results are shown in Table 2, “0” in second column and third column means that the algorithm is not used, and “1” means using the algorithm. When the combination does not adopt the improved coning compensation algorithm (ICCA), the traditional 3-sample coning algorithm is used for attitude calculation. The maximum error means the maximum value of the absolute value of the difference between the attitude resolution and the theoretical value of the trajectory generator.

Table 2 Attitude errors of different combinations of algorithms (pitching angle)

Combination	ATFPF	ICCA	Mean（rad）	Variance（rad^2）	Maximum error（rad）
1	0	0	4.8060e-03	0.01528	0.04389
2	0	1	-8.2797e-04	0.00252	0.00554
3	1	0	2.5294e-03	0.01020	0.02908
4	1	1	8.4489e-05	0.00126	0.00129

As shown in Table 2, the combination 4 which adopts ATFPF and improved coning compensation algorithm has the smallest mean, variance of error and maximum error in all combinations. Compared with combination 1 which is not adopt either algorithm, the error mean of combination 4 can be decreased by about one order of magnitude. The combination of one method used alone can reduce the error, but the effect is worse than the combination 4. This shows that the combination of the two algorithms can improve the accuracy of SINS in vibration environment.

After attitude calculation by combination 4, taking the pitch angle as an example, the result is compared with the theoretical value as shown in Fig. 4. Errors of attitude angles are shown in Fig. 5,

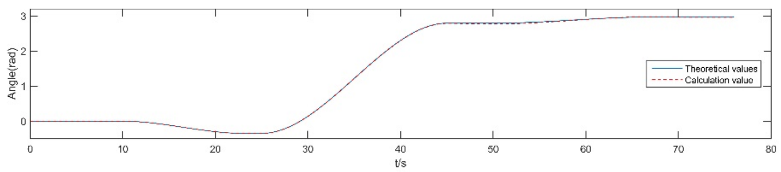

Fig. 4 Comparison of pitch angle

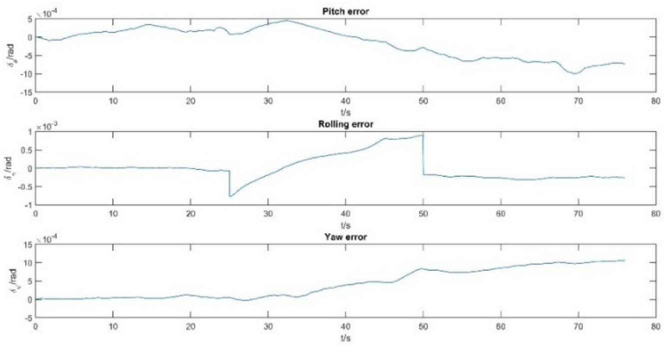

Fig. 5 Error of attitude angles

As shown in Fig. 4 and Fig. 5, after ATFPF filtering and the compensation of improved three-sample coning algorithm, the calculation results of SINS are close to the theoretical value of trajectory generator. The order of maximum error is $10^{-3}\mathrm{rad}(\approx 0.057°)$. It illustrates that this method can suppress noise to some extent and compensate for the attitude error caused by vibration.

5. Conclusions

Aiming at the compensation and suppression of vibration error in SINS, a method based on ATFPF and improved coning compensation algorithm is proposed in this paper. The effectiveness of the method is verified by simulation experiments and the navigation accuracy of SINS is improved.

Acknowledgments

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